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Conformal Cosmology and Supernova Data

Danilo Behnke, David Blaschke
Department of Physics, University of Rostock, 18051 Rostock, Germany

Victor Pervushin, Denis Proskurin
Bogolubov Laboratory for Theoretical Physics, Joint Institute for Nuclear Research
141980 Dubna, Russia

Abstract

We define the cosmological parameters $H_{c,0}$, $\Omega_{m,c}$ and $\Omega_{\Lambda,c}$ within the Conformal Cosmology as obtained by the homogeneous approximation to the conformal-invariant generalization of Einstein's General Relativity theory. We present the definitions of the age of the universe and of the luminosity distance in the context of this approach. A possible explanation of the recent data from distant supernovae Ia without a cosmological constant is presented.

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INTRODUCTION

Now there is a very interesting situation in the modern observational cosmology stimulated by new data on the distance-redshift relation published by the supernova cosmology project (SCP) [1] and on the large-scale structure of the microwave background radiation [2]. The SCP data point to an accelerated expansion of the universe and have stimulated new developments within the standard cosmological model. The old naive version of this model with the dust dominance was not sufficient to explain these new data. New fits of the modern data in the framework of the standard Friedmann-Robertson-Walker (FRW) model were forced to introduce a nonvanishing Λ-term [3]. The occurence of this term has also been interpreted as due to a new form of matter called "quintessence" [3]. What is the origin of the "quintessence" and why does its density approximately coincide with that of matter (luminous plus dark one) at the present stage? Present theories - containing a scalar field - can describe the data by fitting its effective potential [4] but cannot answer these questions.

One of the interesting alternatives to the standard FRW cosmological model is the Jordan-Brans-Dicke (JBD) scalar-tensor theory [5,6] with two homogeneous degrees of freedom, the scalar field and the scale factor. Another alternative is the conformal-invariant version of General Relativity (GR) based on the scalar dilaton field and the geometry of similarity (following Weyls ideas [7]) developed in [5,8–11]. This dilaton version of GR (considered also as a particular case of the Jordan-Brans-Dicke scalar tensor theory of gravitation [12]) is the basis of some speculations on the unification of Einstein's gravity with the Standard Model of electroweak and strong interactions [8,10,13] including modern theories of supergravity [13]. In the conformal-invariant Lagrangian of matter, the dilaton scales the masses of the elementary particles in order to conserve scale invariance of the theory. However, in the current literature [13] a peculiarity of the conformal-invariant version of Einstein's dynamics has been overlooked. The conformal-invariant version of Einstein's dynamics is not compatible with the absolute standard of measurement of lengths and times given by an interval in the Riemannian geometry as this interval is not conformal-invariant. As it has been shown by Weyl in 1918, conformal-invariant theories correspond to the relative standard of measurement of a conformal-invariant ratio of two intervals, given in the geometry of similarity as a manifold of Riemannian geometries connected by conformal transformations [7]. The geometry of similarity is characterized by a measure of changing the length of a vector in its parallel transport. In the considered dilaton case, it is the gradient of the dilaton Φ [10]. In the following, we call the scalar conformal-invariant theory the conformal general relativity (CGR) to distinguish it from the original Weyl theory where the measure of changing the length of a vector in its parallel transport is a vector field (that leads to the defect of the physical ambiguity of the arrow of time pointed out by Einstein in his comment to Weyl's paper [7]). In the present paper we will apply this approach to a description of the Hubble diagram (m(z)-relation) including recent data from the SCP [1] at $z \sim 1$. We make a prediction for the behaviour at $z > 1$ which drastically deviates from that of the standard FRW cosmology with a Λ - term. We suggest this as a test which could discriminate between alternative cosmologies when new data are present in near future.

The present paper is devoted to the definition of the cosmological parameters in the Conformal Cosmology by the analogy with the standard cosmological model [14]. To emphasize the mathematical equivalence of both cases we try to repeat the standard model definitions

restricting ourselves by the consideration of the dust-, curvature-, and Λ-terms.

THEORY AND GEOMETRY

We start from the conformal-invariant theory described by the sum of the dilaton action and the matter action

$$W = W_{\rm CGR} + W_{\rm matter}. \tag{1}$$

The dilaton action is the Penrose-Chernikov-Tagirov one for a scalar (dilaton) field with the negative sign

$$W_{\rm CGR}(g|\Phi) = \int d^4x \left[-\sqrt{-g}\frac{\Phi^2}{6} R(g) + \Phi \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) \right] \; . \tag{2}$$

The conformal-invariant action of the matter fields can be chosen in the form

$$W_{\rm matter} = \int d^4x \left[\mathcal{L}_{(\Phi=0)} + \sqrt{-g}(-\Phi F + \Phi^2 B - \lambda \Phi^4) \right] \; , \tag{3}$$

where B and F are the mass contributions to the Lagrangians of the vector (v) and spinor (ψ) fields, respectively,

$$B = v_i (y_v)_{ij} v_j \; ; \quad F = \bar{\psi}_\alpha (y_s)_{\alpha\beta} \psi_\beta \; , \tag{4}$$

with $(y_v)_{ij}$, and $(y_s)_{\alpha\beta}$ being the mass matrices of vector bosons and fermions coupled to the dilaton field. The massless part of the Lagrangian density of the considered vector and spinor fields is denoted by $\mathcal{L}_{(\Phi=0)}$. The class of theories of the type (1) includes the superconformal theories with supergravity [13] and the standard model with a massless Higgs field [10] as the mass term would violate the conformal symmetry.

HOMOGENEOUS APPROXIMATIONS

In the Conformal Cosmology, the evolution of a universe is described by the scalar dilaton field which can be decomposed into a homogeneous, time dependent component and fluctuations, $\Phi(T,x) = \varphi(T) + \chi(T,x)$. In the homogeneous approximation we neglect the fluctuations and start with the line element of a homogeneous and isotropic universe, which is described by the conformal version of the Friedmann-Robertson-Walker (FRW) metric without the scale factor, as it disappears due to conformal invariance:

$$(ds)^2_c = g_{00}(t) dt^2 - \left[\frac{dr^2}{1 - k_c r^2/r_0^2} + r^2 \Big(d\theta^2 + \sin^2\theta \, d\phi^2 \Big) \right] \; . \tag{5}$$

We define

$$dT = \sqrt{g_{00}} dt \tag{6}$$

as the conformal time interval. From the constraint-type equation $\delta W/\delta g_{00} = 0$ we get

$$\varphi'^2 = \rho_c \varphi + \lambda \varphi^4 - \frac{k_c \varphi^2}{r_0^2} = \rho_C \ , \tag{7}$$

see also [15].

There is a direct correspondence between the conformal cosmology and the standard model obtained by the conformal transformations

$$dt_f = \frac{\varphi(T)}{\varphi(T_0)} dT = \frac{a(T)}{a(T_0)} dT \ , \tag{8}$$

$$\rho_f = \frac{\rho_C}{a^4(T)} \ , \tag{9}$$

$$\Lambda = \lambda \varphi(T_0)^4 \ , \tag{10}$$

where $a(T_0) = 1$, $\varphi(T_0) = M_{\text{Planck}} \sqrt{3/(8\pi)}$. The Friedmann time and density are denoted by t_f and ρ_f, respectively, $a(T)$ is the scale factor and T_0 the present value of the conformal time.

DETERMINATION OF THE CONFORMAL COSMOLOGICAL PARAMETERS

We can define the conformal Hubble-constant

$$H_c = \frac{\varphi'}{\varphi} = \frac{1}{\varphi} \frac{d\varphi}{dT} \ . \tag{11}$$

and rewrite (7) to

$$H_c^2(T) = \frac{\rho_c}{\varphi(T)} + \lambda \varphi(T)^2 - \frac{k_c}{r_0^2}. \tag{12}$$

Applying it to the present time ($T = T_0$), we can write

$$1 = \Omega_{m,c} + \Omega_{\Lambda,c} + \Omega_{k,c} \ , \tag{13}$$

with the dimensionless parameters

$$\Omega_{m,c} \equiv \frac{\rho_c}{\varphi_0 H_{c,0}^2} \ ,$$

$$\Omega_{\Lambda,c} \equiv \frac{\lambda \varphi_0^2}{H_{c,0}^2} \ ,$$

$$\Omega_{k,c} \equiv - \frac{k}{r_0^2 H_{c,0}^2} \ , \tag{14}$$

where $H_{c,0}$ is the value of φ'/φ at the present time. So far, we have discussed only one of the independent equations that arises among the set given in Einstein's field equations. In order to proceed, we need the other. This is in fact equivalent to the statement of conservation of

matter, which means that the quantity ρ_c is constant in time, and the present-day value of the dust matter density is

$$\rho_o = \rho_c \varphi_0 \ . \tag{15}$$

From now on we will use dimensionless variables instead of φ and T. We define

$$y \equiv \varphi/\varphi_0 \,, \qquad \tau_c \equiv H_{c,0}(T - T_0) \,. \tag{16}$$

Using these variables, we can rewrite Eq. (12) in the following form:

$$\begin{aligned} \left(\frac{dy}{d\tau_c}\right)^2 &= \frac{1}{H_{c,0}^2}\left[\frac{\rho_c y}{\varphi_0} + \lambda y^4 \varphi_0^2 - \frac{k_c y^2}{r_0^2}\right] \\ &= y^2\left[\frac{1}{y}\Omega_{m,c} + y^2 \Omega_{\Lambda,c} + \Omega_{k,c}\right] . \end{aligned} \tag{17}$$

Eliminating $\Omega_{k,c}$ now using Eq. (13), we obtain

$$\left(\frac{dy}{d\tau_c}\right)^2 = y^2\Big[1 + \Big(\frac{1}{y} - 1\Big)\Omega_{m,c} + \Big(y^2 - 1\Big)\Omega_{\Lambda,c}\Big] \,, \tag{18}$$

or

$$d\tau_c = \frac{dy}{y\sqrt{1 + \left(\frac{1}{y} - 1\right)\Omega_{m,c} + \left(y^2 - 1\right)\Omega_{\Lambda,c}}} \,. \tag{19}$$

If there was a big bang, y was zero at the time of the bang, i.e., at $T = 0$. On the other hand, $y = 1$ now, by definition. Integrating Eq. (19) between these two limits, we obtain

$$H_{c,0} T_0 = \int_0^1 \frac{dy}{y\sqrt{1 + \left(\frac{1}{y} - 1\right)\Omega_{m,c} + \left(y^2 - 1\right)\Omega_{\Lambda,c}}} \,. \tag{20}$$

This is the equation which shows that the age of the universe is *not* independent, but rather is determined by $H_{c,0}$, $\Omega_{m,c}$ and $\Omega_{\Lambda,c}$. For the special case of a flat, dust universe without cosmological term ($\Omega_{m,c} = 1$, $\Omega_{\Lambda,c} = 0$), we have $T_0 = 2\ H_{c,0}^{-1}$.

Conventionally, one does not use the dimensionless parameter y, but rather uses the *red-shift parameter* z, defined by

$$1 + z \equiv \frac{\varphi_0}{\varphi} = \frac{1}{y} \,. \tag{21}$$

Using this variable, Eq. (19) becomes

$$d\tau_c = \frac{dz}{\sqrt{(1+z)^2(1 + \Omega_{m,c} z) - z(2+z)\Omega_{\Lambda,c}}} \,, \tag{22}$$

so that Eq. (20) can be written in the following equivalent form:

$$H_{c,0}T_0 = \int_0^\infty \frac{dz}{\sqrt{(1+z)^2(1+\Omega_{m,c}z) - z(2+z)\Omega_{\Lambda,c}}} \,. \tag{23}$$

Later we will discuss what sort of evolution does this equation represent. In order to discuss the evolution of the universe, let us not integrate Eq. (19) all the way to the initial singularity, but rather to any arbitrary time T. This gives

$$H_{c,0}(T - T_0) = \int_0^y \frac{dy'}{y'\sqrt{1 + \left(\frac{1}{y'} - 1\right)\Omega_{m,c} + \left(y'^2 - 1\right)\Omega_{\Lambda,c}}} \,. \tag{24}$$

Equivalently, using the red-shift variable, we can write

$$H_{c,0}(T_0 - T) = \int_0^z \frac{dz'}{\sqrt{(1+z')^2(1+\Omega_{m,c}z') - z'(2+z')\Omega_{\Lambda,c}}} \,. \tag{25}$$

DISTANCE VS REDSHIFT RELATION

A light ray traces a null geodesic, i.e., a path for which $(ds)_c^2 = 0$ in (5). Thus, a light ray coming to us satisfies the equation

$$\frac{dr}{dT} = \sqrt{1 - k_c r^2/r_0^2} \,, \tag{26}$$

where r_0 is the dimensionless co-ordinate distance introduced in Eq. (5). Using Eqs. (21) and (22), we can rewrite it as

$$\begin{aligned} \frac{dr}{\sqrt{1 + \Omega_{k,c}H_{c,0}^2 r^2}} &= dT \\ &= \frac{1}{H_{c,0}} \frac{dz}{\sqrt{(1+z)^2(1+\Omega_{m,c}z) - z(2+z)\Omega_{\Lambda,c}}} \,, \end{aligned} \tag{27}$$

where on the left side, we have replaced k_c by $\Omega_{k,c}$ using the definition of Eq. (14). Integration of this equation determines the co-ordinate distance as a function of z:

$$\begin{aligned} H_{c,0}r(z) = &\frac{1}{\sqrt{|\Omega_{k,c}|}} \times \\ &\text{sinn}\left[\sqrt{|\Omega_{k,c}|}\int_0^z \frac{dz'}{\sqrt{(1+z')^2(1+\Omega_{m,c}z') - z'(2+z')\Omega_{\Lambda,c}}}\right] \,, \end{aligned} \tag{28}$$

where $\text{sinn}(x) = \sinh(x)$ for $\Omega_{k,c} > 0$, $\text{sinn}(x) = \sin(x)$ for $\Omega_{k,c} < 0$ and $\text{sinn}(x) = x$ for the flat universe with $\Omega_{k,c} = 0$. The equation (28) coincides with the similar relation between the coordinate distance and the redshift in Standard Cosmology [14]. The physical distance to a certain object can be defined in various ways. For what follows, we will need what is

called the "luminosity distance" ℓ_f, which is defined in a way that the apparent luminosity of any object goes like $1/\ell_f^2$. In Standard Cosmology we have

$$\ell_f(z) = a_0^2 r(z)/a(z) = (1+z) a_0 r \,. \tag{29}$$

Thus,

$$\begin{aligned} H_0\,\ell_f(z) = {} & \frac{1+z}{\sqrt{|\Omega_k|}} \times \\ & \mathrm{sinn}\left[\sqrt{|\Omega_k|}\int_0^z \frac{dz'}{\sqrt{(1+z')^2(1+\Omega_m z') - z'(2+z')\Omega_\Lambda}}\right]. \end{aligned} \tag{30}$$

Any observable distances $\ell_f(z)$ in the Standard Cosmology can be converted into observable distances $\ell_c(z)$ in the Conformal Cosmology by the conformal transformation

$$d\ell_c = (1+z) d\ell_f(z) \ . \tag{31}$$

Considering the flat universe with $\Omega_\Lambda = 0$ in the dust stage, we get in the Standard Cosmology

$$\ell_f(z) = 2[(1+z) - \sqrt{1+z}] \ . \tag{32}$$

For comparison we obtain for the dust case in the Conformal Cosmology

$$\ell_c(z) = (2z + z^2) - 2/3[(1+z)^{3/2} - 1] \ , \tag{33}$$

see Fig. 1. For plotting we used the well known $m(z)-$relation given by $m(z) = 5\log[H_0 \ell(z)] + \mathcal{M}$, where $\mathcal{M}$ is a constant.

CONCLUSION

We have defined the cosmological parameters $H_{c,0}$, $\Omega_{m,c}$ and $\Omega_{\Lambda,c}$ in the Conformal Cosmology. We have shown how the age of the universe depends on them. The important result of the above derivation of the cosmological parameters in the Conformal Cosmology is the fact, that we can fit the recent Supernova data at $z \sim 1$ in the Hubble diagram (m(z)-relation) in a simple dust case very well and therefore do not need a cosmological constant. Furthermore we gave a prediction for the behaviour at $z > 1$ which deviates from the standard FRW cosmology with a non-vanishing Λ-term. Within this model we suggest as a possible explanation of the recent data from the Supernova Cosmology Project a static (nonexpanding) flat universe where the apparent "acceleration" stems from the evolution of the scalar dilaton field in Conformal General Relativity, when applied to cosmology.

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FIGURES

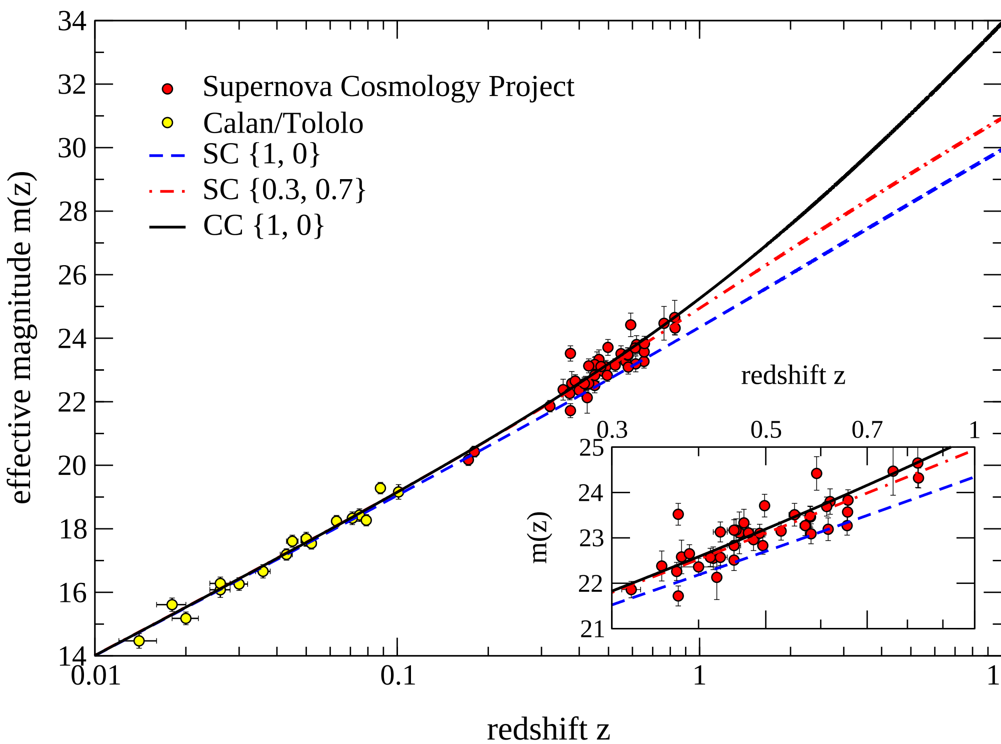

FIG. 1. $m(z)$- relation for a flat universe model in Standard and Conformal Cosmology. The data points include those from 42 high-redshift Type Ia supernovae [1]. An optimal fit to these data within the Standard Cosmology requires a cosmological constant $\Omega_\Lambda = 0.7$, whereas in the Conformal Cosmology presented here no cosmological constant is needed.